# J-integral analysis of the strain fields of micro-cracks in single silicon crystal using HR-EBSD

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**Abstract:** A novel method has been developed to quantify the stress and strain fields at microscale stress concentrations, which is demonstrated in an analysis of a cleavage micro-crack in single crystal silicon. The method utilises a finite-element based J-integral analysis, which employs the elastic strain field measured by High-Resolution Electron Backscatter Diffraction (HR-EBSD). This method can be applied to micro-cracks in anisotropic materials and ultimately is to be used in in situ experiments that quantify the critical strain energy release rate for micro-crack propagation.

## 1. Introduction

To understand how the structural properties (i.e. strength, ductility and toughness) of engineering metals and alloys can be improved, it is critical to study the criteria for propagation and the interactions between deformation-induced features (DIFs) (i.e. deformation twins, micro-cracks and slip bands) and microstructural features (e.g. grain boundaries and other DIFs), as these can produce local strain concentrations that may initiate fracture. This knowledge is needed for microstructure-informed design of materials using modelling tools such as crystal plasticity. Thus, we need high-resolution data that characterise the stress and strain fields at, or approaching, a critical state. This may be approached by ex situ measurements; although, in-situ studies are the ultimate aim because ex-situ measurements can be affected by stress relaxation.

Previous work at Oxford on the full-field analysis of the elastic strain fields around cracks has shown that elastic strain maps obtained by synchrotron X-ray diffraction <sup>1–3</sup> can be analysed via a finite-element based method to quantify the elastic strain energy field via the J-integral <sup>4,5</sup>. Hence, the potential strain energy release rate for crack propagation can be quantified by local measurements without knowledge of the external boundary conditions (i.e. load, crack length). However, these studies were done at a relatively large scale (cm-size specimens) on long cracks with low spatial resolution.

Previous studies at the microscale of slip bands and twins using HR-EBSD (High-resolution Electron Backscatter Diffraction) <sup>6–8</sup> and differential aperture X-ray Laue micro-diffraction (DAXM) <sup>9,10</sup> have shown their elastic strain fields may be measured, and the strain energy release rate quantified as a stress intensity factor by fitting a line profile to the stress field. We have developed a more robust method that use the full field measurements <sup>11</sup> to quantify the strain energy fields of general DIFs (cracks, slip bands and twins) in the microstructure of engineering alloys, which uses a Finite Element J-integral analysis of HR-EBSD data. This method can open the way to in situ studies, including propagation of short fatigue cracks, deformation-twins and slip-bands, as well as environment-assisted cracking and intergranular fracture.

### 2. Methods and Results

High (angular) resolution electron backscattering diffraction (HR-EBSD) is a very convenient non-destructive method to obtain the elastic deformation field as it quantifies elastic strain and lattice rotations with a sensitivity of  $\pm 10^{-4}$  <sup>12,13</sup>. The elastic strain field can then be integrated to solve for the displacement field using the finite element method (FEM). The integration method employs FEM's shape function and the Jacobian for a robust solution of uniform and non-uniform measurements, in contrast to the previous method<sup>1</sup> that used nodal displacements. The algorithm also uses recently developed MATLAB algorithms (i.e. Isqminnorm) to efficiently invert dense and sparse matrices.

Single-crystal Silicon has been studied as a model brittle material. An example analysis of a {110} cleavage crack, induced by indentation on the {001} plane is presented in Figure 1, which shows maps of the in-plane normal and shear strains and the displacement field inferred from these using knowledge of the crystal elastic constants. The mode-I and mode-II stress intensity factors have been extracted from the J-integral via the interaction integral. The final result is relatively contour insensitive. It shows the crack tip is mixed-mode loaded, with a mode I stress intensity factor that is smaller than the mode-I fracture toughness for {110} cleavage. This is due to stress relaxation after indentation. Work is now in progress to study stable crack propagation via in situ observations. This is challenging, due to practical issues related to SEM stage drift and limitations in the speed of data acquisition with current detectors. We aim to extend the work to study non-cleavage cracks, deformation twins and slip-bands.

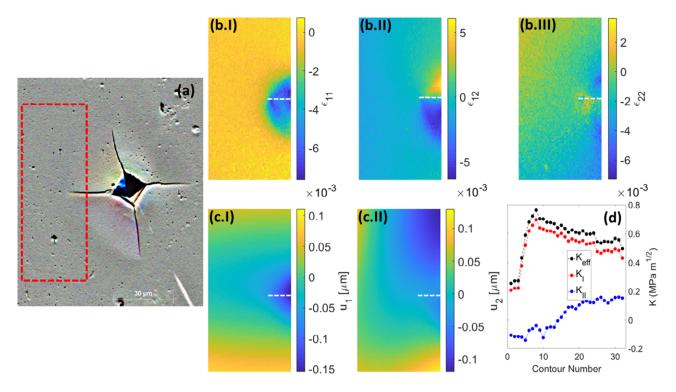


Figure 1: Ex situ characterisation via HR-EBSD of (a) a {110} cleavage crack in single crystal silicon. b) the elastic strain components for i)  $\varepsilon_{11}$ , ii)  $\varepsilon_{12}$  and iii)  $\varepsilon_{22}$ ; c) the integrated displacement field for the normal components i) U<sub>1</sub> and ii) U<sub>2</sub>; c) the mode I and mode II stress integrity factors extracted from the J-integral analysis as a function of contour number from the crack tip highlighted with white dotted line.

#### 3. Conclusion

We presented a novel method to quantify the field ahead of micro-cracks using HR-EBSD measurement of the elastic strain field. A finite element method-based integrates the elastic strain field to obtain the displacement field, from which the full-field J-integral provides the strain energy release rate. The mode I and mode II stress intensity factors are then obtained using the interaction integral.

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